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Suppression of nonlinear vibration system described by nonlinear differential equations using passive controller

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Abstract In this paper, the linear absorber is proposed to reduce the vibration of a nonlinear dynamical system at simultaneous primary resonance and the presence of 1:1 internal resonance. This leads to a two-degree-of-freedom system subjected to external excitation force. The method of multiple scales perturbation technique is applied throughout to determine the analytical solution up to first-order approximations. The stability of the system near the one of the worst resonance case is studied using the frequency response equations. The effects of the different system and absorber parameters on the behavior of the main system are studied numerically. For validity, the numerical solution is compared with the analytical solution and gets a good agreement. Effectiveness of the absorber (E_a) is about 800 for the nonlinear vibrating system. The simulation results are achieved using MATLAB programs. At the end of the work, the comparison with the available published work is reported.

Keywords Passive vibration control · Absorber · Nonlinear dynamical system · Stability · Multiple scales · Simultaneous resonance

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1 Introduction

There are several strategies to reduce the vibration which occurring in the most important dynamical systems such as linear and nonlinear time-delayed position feedback, passive controller (absorbers), the nonlinear saturation controller (NSC), the positive position feedback (PPF) controller, cubic feedback control laws, and the velocity feedback control law. Warminski et al. [1] illustrated the numerical and experimental studies for different types of active controllers applied to nonlinear beam models. The results for a single-beam system show that NSC and PPF controllers are the most effective for assumed conditions of the plant.

El-Ganaini et al. [2] studied the positive position feedback controller to suppress the vibration amplitude of a nonlinear dynamic model at primary resonance and the presence of 1:1 internal resonance. Saeed et al. [3] presented the nonlinear time delay saturation-based controller for the simultaneously resonance case ($\Omega \cong \omega_1$ and $\omega_1 \cong \omega_2$) of a dynamical system. It is seen that the vibration can be suppressed at some values of time delay. These values form a so called "vibration suppression region" which we found as a periodic function of time delay. Yaman and Sen [4] show that the primary structure consists of a flexible beam which has a single degree of freedom, and is subjected to a vertical sinusoidal base excitation. The primary objective of this study is to determine the effectiveness of pendulum-type passive vibration absorber attached to a primary structure whose orientation varies. Eissa and

Amer [5] controlled the vibration of the first mode of the cantilever beam subjected to a primary and sub-harmonic resonance ($\Omega_1 \cong 1$ and $\Omega_2 \cong 2$) by adding a feedback cubic nonlinear term $\left(T = Gx^3\right)$, which can be used to control the amplitude of the system to 33 %. Oueini et al. [6] studied a nonlinear absorber based on saturation phenomena as a vibration absorber for a linear model of a cantilever beam. The influences of damping of the controller and loops gains have been shown using multiple scale method. Also, the sensitivity on the initial conditions has been checked too. The positive position feedback (PPF) controller applied for a flexible manipulator is presented by Shan et al. [7]. They presented several vibration modes in the control strategy taking into account a linear mathematical model of the plant. The PPF control has been compared with the algorithm of velocity feedback. An experimental study shows that only PPF algorithm is able to work properly, while slewing process is realized. Hegazy [8] studied the dynamic behavior and chaotic motion of a string-beam coupled system subjected to parametric excitation. The case of 3:1 internal resonance between the modes of the beam and string in the presence of sub-harmonic resonance for the beam was considered and examined.

Zhang et al. [9] investigated the chaotic dynamics and bifurcations of a simply supported symmetric cross-ply composite laminated piezoelectric rectangular plate subject to the transverse, in-plane excitations, and the excitation loaded by piezoelectric layers. The case of 1:2 internal resonance and primary parametric resonance is considered.

Refs. [10–17] applied different type of controllers as passive and active control to reduce the vibration of the nonlinear dynamical system.

In this paper, we modified the work of Refs [1,2] and introduced the linear absorber to suppress the vibration of a nonlinear dynamical system subjected to external excitation. MSPT to the first-order approximations is applied to analyze the response of the modified system near the simultaneous primary and internal resonance to obtain the semi-closed form solution. The stability of the system is investigated near this worst simultaneous resonance case applying frequency response equations. Some recommendations regarding the different parameters of the system are reported and the effect of the absorber on system behavior is given numerically. Comparison with the available published work is reported.

2 Nonlinear dynamical system model

The investigated nonlinear ordinary differential equation which described the nonlinear dynamical system [1,2] is given as

$$\begin{aligned}&\ddot{u} + 2\mu\omega_1\dot{u} + \omega_1^2 u + \beta u^3 - \delta(u\dot{u}^2 + u^2\ddot{u})\\&\quad + \zeta(\dot{u} - \dot{v}) + \alpha(u - v) = f\cos(\Omega t)\end{aligned} \tag{1}$$

Here, we introduced a linear tuned mass absorber which connected to the main system through a control law. Then the modified equations of motion are two-coupled differential equations, one nonlinear and one linear, by the same method in [18] as the following:

$$\begin{aligned}&\ddot{u} + 2\mu\omega_1\dot{u} + \omega_1^2 u + \beta u^3 - \delta(u\dot{u}^2 + u^2\ddot{u})\\&\quad + \zeta(\dot{u} - \dot{v}) + \alpha(u - v) = f\cos(\Omega t)\end{aligned} \tag{2}$$

$$\ddot{v} + 2\zeta_1\omega_2(\dot{v} - \dot{u}) + \omega_2^2(v - u) = 0 \tag{3}$$

where u and v are independent generalized coordinates correspond to the two degrees of the system, μ, ζ, and ζ_1 are damping coefficient of the system and absorber, f and Ω are amplitude and frequency of external excitation force applied to the system by forte object, α is a linear parameter, β and δ are nonlinear parameters, ω_1 and ω_2 are natural frequencies of the system and the absorber, respectively, where $u(0) = 0$, $\dot{u}(0) = 0$, $v(0) = 0$, and $\dot{v}(0) = 0$.

Suppose that $\mu = \varepsilon\hat{\mu}$, $\beta = \varepsilon^{-1}\hat{\beta}$, $\delta = \varepsilon^{-1}\hat{\delta}$, $f = \varepsilon^2\hat{f}$, $\zeta = \varepsilon\hat{\zeta}$, $\zeta_1 = \varepsilon\hat{\zeta}_1$, $\alpha = \varepsilon\hat{\alpha}$ where ε is a small dimensionless book-keeping perturbation parameter and $0 < \varepsilon \ll 1$. Then the government equations for the dynamical system with the absorber can be written as

$$\begin{aligned}&\ddot{u} + 2\varepsilon\hat{\mu}\omega_1\dot{u} + \omega_1^2 u + \varepsilon^{-1}\hat{\beta}u^3 - \varepsilon^{-1}\hat{\delta}(u\dot{u}^2 + u^2\ddot{u})\\&\quad + \varepsilon\hat{\zeta}(\dot{u} - \dot{v}) + \varepsilon\hat{\alpha}(u - v) = \varepsilon^2\hat{f}\cos(\Omega t)\end{aligned} \tag{4}$$

$$\ddot{v} + 2\varepsilon\hat{\zeta}_1\omega_2(\dot{v} - \dot{u}) + \omega_2^2(v - u) = 0 \tag{5}$$

2.1 Mathematical analysis (multiple scales method)

Applying MSPT [19,20] is used to obtain a uniformly valid, asymptotic expansion of the solutions for Eqs. (4)–(5)

The asymptotic approximate solutions of Eqs. (4)–(5) are assumed in the forms:

$$u(t,\varepsilon) = \varepsilon u_1(T_0,T_1) + \varepsilon^2 u_2(T_0,T_1) + O(\varepsilon^3) \tag{6}$$

$$v(t,\varepsilon) = \varepsilon v_1(T_0,T_1) + \varepsilon^2 v_2(T_0,T_1) + O(\varepsilon^3) \tag{7}$$

The derivatives will be in the form:

$$\frac{\mathrm{d}}{\mathrm{d}t} = \frac{\mathrm{d}T_0}{dt}\frac{\partial}{\partial T_0} + \frac{\mathrm{d}T_1}{\mathrm{d}t}\frac{\partial}{\partial T_1} + \cdots = D_0 + \varepsilon D_1 + \cdots \tag{8}$$

$$\frac{\mathrm{d}^2}{\mathrm{d}t^2} = D_0^2 + 2\varepsilon D_0 D_1 + \cdots \tag{9}$$

where $T_n = \varepsilon^n t$ and $D_n = \partial/\partial T_n \quad (n = 0, 1)$.

Substituting Eqs. (6)–(9) into Eqs. (4)–(5) and equating the coefficients of equal power of (ε) lead to

$O(\varepsilon^1)$

$$\left(D_0^2 + \omega_1^2\right) u_1 = 0 \tag{10a}$$

$$\left(D_0^2 + \omega_2^2\right) v_1 = \omega_2^2 u_1 \tag{10b}$$

$O(\varepsilon^2)$

$$\left(D_0^2 + \omega_1^2\right) u_2 = -2D_0 D_1 u_1 - 2\hat{\mu}\omega_1 D_0 u_1 - \hat{\beta} u_1^3 + \hat{\delta}\left[u_1 (D_0 u_1)^2 + u_1^2 D_0^2 u_1\right] - \hat{\zeta}\left[D_0 u_1 - D_0 v_1\right] - \hat{\alpha}\left[u_1 - v_1\right] + \hat{f}\cos(\Omega t) \tag{11a}$$

$$\left(D_0^2 + \omega_2^2\right) v_2 = -2D_0 D_1 v_1 - 2\hat{\zeta}_1 \omega_2 \left[D_0 v_1 - D_0 u_1\right] + \omega_2^2 u_2 \tag{11b}$$

The general solution of Eq. (10) represents the zeroth order approximation and can be written in the form:

$$u_1(T_0, T_1) = A_1(T_1)\exp(i\omega_1 T_0) + cc. \tag{12}$$

$$v_1(T_0, T_1) = A_2(T_1)\exp(i\omega_2 T_0) + \Gamma A_1(T_1)\exp(i\omega_1 T_0) + cc. \tag{13}$$

Where the quantities $A_1(T_1)$ and $A_2(T_1)$ are unknown function in T_1 at this stage of the analysis , cc represents the complex conjugate of the previous terms and $\Gamma = \frac{\omega_2^2}{(\omega_2^2 - \omega_1^2)}$.

Substituting Eqs. (12)–(13) into Eq. (11) and eliminating the secular terms, then the general solution of Eq. (11) which represents the first-order approximation is obtained as follows:

$$u_2 = B_1 \exp(i\omega_1 T_0) + \left[\frac{\hat{\beta} A_1^3 + 2\hat{\delta}\omega_1^2 A_1^3}{8\omega_1^2}\right]\exp(3i\,\omega_1 T_0) + \left[\frac{i\omega_2\hat{\zeta} A_2 + \hat{\alpha} A_2}{(\omega_1^2 - \omega_2^2)}\right]\exp(i\omega_2 T_0) + \frac{\hat{f}}{2(\omega_1^2 - \Omega^2)}\exp(i\,\Omega T_0) + cc. \tag{14}$$

$$v_2 = B_2 \exp(i\omega_2 T_0) + \left[\frac{-2i\omega_1 D_1 \Gamma A_1 - 2i\hat{\zeta}_1\omega_1\omega_2(\Gamma - 1)A_1}{(\omega_2^2 - \omega_1^2)}\right]\exp(i\,\omega_1 T_0) + \left[\frac{\omega_2^2\left(\hat{\beta} A_1^3 + 2\hat{\delta}\omega_1^2 A_1^3\right)}{8\omega_1^2(\omega_2^2 - 9\omega_1^2)}\right]\exp(3i\,\omega_1 T_0) + \left[\frac{\omega_2^2 \hat{f}}{2(\omega_1^2 - \Omega^2)(\omega_2^2 - \Omega^2)}\right]\exp(i\,\Omega T_0) + cc. \tag{15}$$

where the quantities $B_1(T_1)$ and $B_2(T_1)$ are unknown function in T_1 at this stage of the analysis , cc represents the complex conjugate of the previous terms.

Then we can get the analytical solution of Eqs. (4)–(5) by substituting Eqs. (12)–(15) into Eqs. (6)–(7).

3 Stability using frequency response equation

In this work, the simultaneous primary and internal resonance case $(\Omega \cong \omega_1, \omega_2 \cong \omega_1)$, which is one of the worst resonance case (confirmed numerically), has been chosen to study the stability, of the system of Eqs. (4)–(5), at the first-order approximation. Introducing the detuning parameters σ_1 and σ_2 $\left(\sigma_m = \varepsilon\hat{\sigma}_m\right)$ according to

$$\Omega \cong \omega_1 + \varepsilon\hat{\sigma}_1, \omega_2 \cong \omega_1 + \varepsilon\hat{\sigma}_2 \tag{16}$$

Substituting Eq. (16) into Eq. (11) and eliminating the secular terms lead to the solvability conditions for the first-order approximation, hence the following differential equations are obtained:

$$2i\,\omega_1 D_1 A_1 = \left[-2i\,\hat{\mu}\,\omega_1^2 A_1 - 3\hat{\beta} A_1^2 \bar{A}_1 - 2\hat{\delta}\omega_1^2 A_1^2 \bar{A}_1 - i\omega_1\hat{\zeta} A_1 - \hat{\alpha} A_1\right] + \frac{\hat{f}}{2}\exp\left(i\hat{\sigma}_1 T_1\right) + \left[i\omega_2\hat{\zeta} A_2 + \hat{\alpha} A_2\right]\exp\left(i\hat{\sigma}_2 T_1\right) \tag{17}$$

$$2i\omega_2 D_1 A_2 = \left[-2i\hat{\zeta}_1\omega_2^2 A_2\right] + \left[2i\hat{\zeta}_1\omega_1\omega_2 A_1\right]\exp\left(-i\,\hat{\sigma}_2 T_1\right) \tag{18}$$

From Eq. (8) and multiplying both sides by $2i\omega_m (m = 1, 2)$, we can express the derivative of $A_m(T_1)$ with respect to t as

$$2i\,\omega_m \frac{\mathrm{d}A_m}{\mathrm{d}t} = \varepsilon 2\omega_m i D_1 A_m + O(\varepsilon^2) \tag{19}$$

To analyze the solution of Eqs. (17) and (18), it is convenient to express A_m in the polar form

$$A_m(T_1) = \frac{1}{2}\hat{a}_m \exp(i\gamma_m) \qquad a_m = \varepsilon\hat{a}_m \tag{20}$$

where a_m and γ_m are real functions of T_1. Inserting Eqs. (17), (18), and (20) into Eq. (19) and equating the real and imaginary parts yield

$$\dot{a}_1 = \left[-\mu\omega_1 a_1 - \frac{\zeta}{2}a_1\right] + \left[\frac{\omega_2\zeta}{2\omega_1}a_2\right]\cos(\theta_2) + \left[\frac{\alpha}{2\omega_1}a_2\right]\sin(\theta_2) + \frac{f}{2\omega_1}\sin(\theta_1) \quad (21)$$

$$\dot{\gamma}_1 a_1 = \left[\frac{3\beta}{8\omega_1}a_1^3 + \frac{\delta\omega_1}{4}a_1^3 + \frac{\alpha}{2\omega_1}a_1\right] - \left[\frac{\alpha}{2\omega_1}a_2\right]\cos(\theta_2) + \left[\frac{\omega_2\zeta}{2\omega_1}a_2\right]\sin(\theta_2) - \frac{f}{2\omega_1}\cos(\theta_1) \quad (22)$$

$$\dot{a}_2 = [-\zeta_1\omega_2 a_2] + [\zeta_1\omega_1 a_1]\cos(\theta_2) \quad (23)$$

$$\dot{\gamma}_2 a_2 = -[\zeta_1\omega_1 a_1]\sin(\theta_2) \quad (24)$$

where $\theta_1 = \hat{\sigma}_1 T_1 - \gamma_1 = \sigma_1 T_0 - \gamma_1$, $\theta_2 = \hat{\sigma}_2 T_1 + \gamma_2 - \gamma_1 = \sigma_2 T_0 + \gamma_2 - \gamma_1$.

The steady-state solution of our dynamical system corresponding to the fixed point of Eqs. (21)–(24) is obtained when $\dot{a}_m = 0$ and $\dot{\theta}_m = 0$, then we get the frequency response equations (FRE) for the practical case ($a_1 \neq 0$, $a_2 \neq 0$) as follows:

$$\left[\mu\omega_1 a_1 + \frac{\zeta}{2}a_1\right] = \left[\frac{\omega_2\zeta}{2\omega_1}a_2\right]\cos(\theta_2) + \left[\frac{\alpha}{2\omega_1}a_2\right]\sin(\theta_2) + \frac{f}{2\omega_1}\sin(\theta_1) \quad (25)$$

$$\sigma_1 a_1 - \left[\frac{3\beta}{8\omega_1}a_1^3 + \frac{\delta\omega_1}{4}a_1^3 + \frac{\alpha}{2\omega_1}a_1\right] = -\left[\frac{\alpha}{2\omega_1}a_2\right]\cos(\theta_2) + \left[\frac{\omega_2\zeta}{2\omega_1}a_2\right]\sin(\theta_2) - \frac{f}{2\omega_1}\cos(\theta_1) \quad (26)$$

$$[\zeta_1\omega_2 a_2] = [\zeta_1\omega_1 a_1]\cos(\theta_2) \quad (27)$$

$$(\sigma_1 - \sigma_2)\, a_2 = -[\zeta_1\omega_1 a_1]\sin(\theta_2) \quad (28)$$

From Eqs. (27) and (28), we get

$$\cos(\theta_2) = \left[\frac{\omega_2 a_2}{\omega_1 a_1}\right] \quad (29)$$

$$\sin(\theta_2) = \left[-\frac{(\sigma_1 - \sigma_2)\, a_2}{\zeta_1\omega_1 a_1}\right] \quad (30)$$

Squaring and adding Eqs. (29) and (30), we get

$$\zeta_1^2\omega_1^2 a_1^2 = \left[\zeta_1^2\omega_2^2 a_2^2\right] + \left[(\sigma_1 - \sigma_2)^2 a_2^2\right] \quad (31)$$

Inserting (29) and (30) into (25) and (26), we get

$$\left[\mu\omega_1 a_1 + \frac{\zeta}{2}a_1\right] - \left[\frac{\omega_2^2\zeta a_2^2}{2\omega_1^2 a_1}\right] + \left[\frac{\alpha(\sigma_1 - \sigma_2)\, a_2^2}{2\omega_1^2\zeta_1 a_1}\right] = \frac{f}{2\omega_1}\sin(\theta_1) \quad (32)$$

$$\sigma_1 a_1 - \left[\frac{3\beta}{8\omega_1}a_1^3 + \frac{\delta\omega_1}{4}a_1^3 + \frac{\alpha}{2\omega_1}a_1\right] + \left[\frac{\alpha\omega_2 a_2^2}{2\omega_1^2 a_1}\right] + \left[\frac{\omega_2\zeta(\sigma_1 - \sigma_2)\, a_2^2}{2\omega_1^2\zeta_1 a_1}\right] = -\frac{f}{2\omega_1}\cos(\theta_1) \quad (33)$$

Squaring and adding Eqs. (32) and (33), we get

$$\left\{\left[\mu\omega_1 a_1 + \frac{\zeta}{2}a_1\right] - \left[\frac{\omega_2^2\zeta a_2^2}{2\omega_1^2 a_1}\right] + \left[\frac{\alpha(\sigma_1 - \sigma_2)\, a_2^2}{2\omega_1^2\zeta_1 a_1}\right]\right\}^2 + \left\{\sigma_1 a_1 - \left[\frac{3\beta}{8\omega_1}a_1^3 + \frac{\delta\omega_1}{4}a_1^3 + \frac{\alpha}{2\omega_1}a_1\right] + \left[\frac{\alpha\omega_2 a_2^2}{2\omega_1^2 a_1}\right] + \left[\frac{\omega_2\zeta(\sigma_1 - \sigma_2)\, a_2^2}{2\omega_1^2\zeta_1 a_1}\right]\right\}^2 = \frac{f^2}{4\omega_1^2} \quad (34)$$

Then the frequency response equations that used to describe the system are Eqs. (31) and (34).

3.1 Stability of nonlinear solution

To determine the stability of the steady-state solution, one lets

$$a_m = a_{m0} + a_{m1}, \quad \theta_m = \theta_{m0} + \theta_{m1} \quad (35)$$

where a_{m0} and θ_{m0} are the solutions of Eqs. (21)–(24) and $a_{m1} and \theta_{m1}$ are perturbations which are assumed to be small compared with a_0 and θ_0. Substituting Eq. (35) into Eqs. (21)–(24) and keeping only the linear terms in a_{m1} and θ_{m1}, we obtain

$$\dot{a}_{11} = \left[-\mu\omega_1 - \frac{\zeta}{2}\right]a_{11} + \left[\frac{f}{2\omega_1}\cos(\theta_{10})\right]\theta_{11} + \left[\frac{\omega_2\zeta}{2\omega_1}\cos(\theta_{20}) + \frac{\alpha}{2\omega_1}\sin(\theta_{20})\right]a_{21} + \left[-\frac{\omega_2\zeta}{2\omega_1}a_{20}\sin(\theta_{20}) + \frac{\alpha}{2\omega_1}a_{20}\cos(\theta_{20})\right]\theta_{21} \quad (36)$$

$$\dot{\theta}_{11} = \left[\frac{\sigma_1}{a_{10}} - \frac{9\beta a_{10}}{8\omega_1} - \frac{9\delta\omega_1 a_{10}}{8} - \frac{\alpha}{2a_{10}\omega_1}\right]a_{11} + \left[-\frac{f\sin(\theta_{10})}{2a_{10}\omega_1}\right]\theta_{11}$$

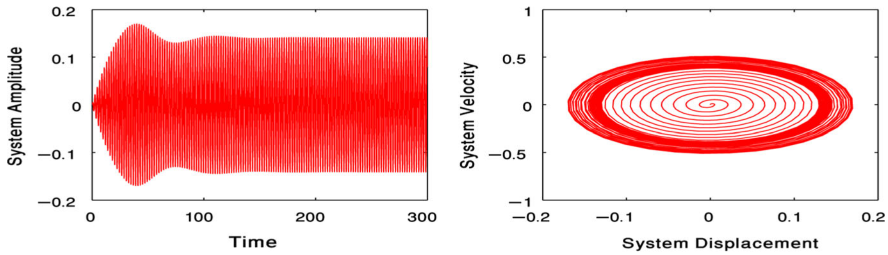

Fig. 1 Time trace of the response and phase-plane diagrams of nonlinear system without absorber for primary resonance case ($\Omega \cong \omega_1$)

$$+\left[-\frac{\omega_2\zeta \sin(\theta_{20})}{2a_{10}\omega_1}+\frac{\alpha\cos(\theta_{20})}{2a_{10}\omega_1}\right]a_{21} + \left[-\frac{\omega_2\zeta a_{20}\cos(\theta_{20})}{2a_{10}\omega_1}-\frac{\alpha a_{20}\sin(\theta_{20})}{2a_{10}\omega_1}\right]\theta_{21} \qquad (37)$$

$$\dot{a}_{21} = [\zeta_1\omega_1\cos(\theta_{20})]\,a_{11} + [-\zeta_1\omega_2]\,a_{21} + [-\zeta_1\omega_1 a_{10}\sin(\theta_{20})]\,\theta_{21} \qquad (38)$$

$$\dot{\theta}_{21} = \left[\frac{\sigma_1}{a_{10}} - \frac{9\beta a_{10}}{8\omega_1} - \frac{9\delta\omega_1 a_{10}}{8} - \frac{\alpha}{2a_{10}\omega_1} - \frac{\zeta_1\omega_1\sin(\theta_{20})}{a_{20}}\right]a_{11} + \left[-\frac{f\sin(\theta_{10})}{2a_{10}\omega_1}\right]\theta_{11} + \left[\frac{(\sigma_2-\sigma_1)}{a_{20}} - \frac{\omega_2\zeta\sin(\theta_{20})}{2a_{10}\omega_1} + \frac{\alpha\cos(\theta_{20})}{2a_{10}\omega_1}\right]a_{21} + \left[-\frac{\omega_2\zeta a_{20}\cos(\theta_{20})}{2a_{10}\omega_1} - \frac{\alpha a_{20}\sin(\theta_{20})}{2a_{10}\omega_1} - \frac{\zeta_1\omega_1 a_{10}\cos(\theta_{20})}{a_{20}}\right]\theta_{21} \qquad (39)$$

The eigenvalues of the above system of equations are given by the equation

$$\lambda^4 + R_1\lambda^3 + R_2\lambda^2 + R_3\lambda + R_4 = 0 \qquad (40)$$

where R_1, R_2, R_3, and R_4 are constants in the parameters ($a_1, a_2, \mu, \zeta, \zeta_1, \sigma_1, \sigma_2, \alpha$, and β, $f, \delta, \omega_1, \omega_2, \theta_1, \theta_2$). If the real part of each eigenvalues is negative, the corresponding equilibrium solution is asymptotically stable and otherwise becomes unstable. According to the Routh–Hurwitz criterion, the necessary and sufficient conditions for all the roots of (40) to possess negative real parts are

$$r_1 > 0,\ r_1r_2 - r_3 > 0,\ r_3(r_1r_2 - r_3) - r_1^2 r_4 > 0 \ \text{ and } \ r_4 > 0.$$

4 Numerical results

In this section, the differential equations of nonlinear dynamical system (main system without and with absorber) at primary and internal resonance cases are solved numerically applying Rung–Kutta fourth-order method using MATLAB 7.14.0.739 (R2012a) package (ode45) at the selected values ($\mu = 0.01, \omega_1 = 3, \delta = 1, \beta = 14.5, \quad f = 0.05, \Omega = \omega_1, \zeta_1 = 0.001, \omega_2 = \omega_1, \alpha = 0.9, \zeta = \zeta_1\omega_2/\zeta_1\omega_2 5.5$).

It can be seen from Fig. 1 that the steady-state amplitude of the main system (u) without absorber is about 320 % of the excitation force amplitude f with slight chaotic limit cycles. The steady-state amplitude of the main system (u) is decreasing to about 0.4 % of the excitation force amplitude f when we added the absorber, and the steady-state amplitude of the absorber (v) is about 120 % of the excitation force amplitude f as shown in Fig. 2.

It is worth to notice that from the Figs. 1 and 2 that the steady-state amplitude of the main system with the absorber was suppressed by about 99.875 % from its value without absorber. This means that the effectiveness of the absorber E_a (E_a = steady-state amplitude of the main system without absorber / steady-state amplitude of the main system with absorber) is about 800 for the main system (u).

4.1 Comparison between analytical and numerical solution

Figures 3 and 4 represent the comparison between analytical solution given by Eqs. (21–24) and the numerical solution of Eqs. (2–3) for nonlinear dynamical system with absorber for chosen values of the system parameters that are presented graphically in Fig. 2. The dashed lines show the modulation of the amplitude for the generalized coordinates u and v. However, the continuous lines represent the time history of vibrations which were obtained numerically as solution of the original equations of the nonlinear dynamical system with absorber.

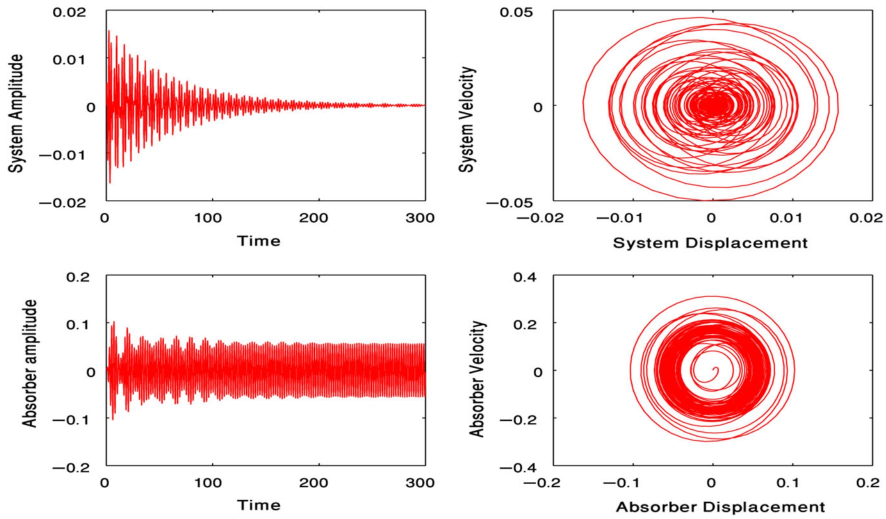

Fig. 2 Time trace of the response and phase-plane diagrams of nonlinear system with absorber for Primary and internal resonance case ($\Omega \cong \omega_1, \omega_2 \cong \omega_1$)

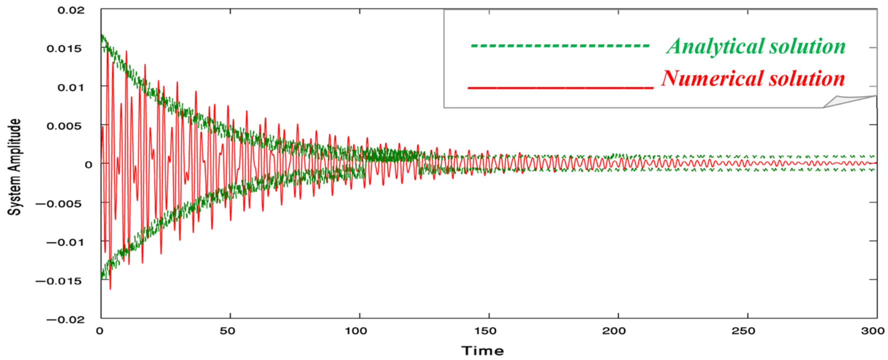

Fig. 3 Comparison between the analytical and numerical solutions of the nonlinear system with absorber

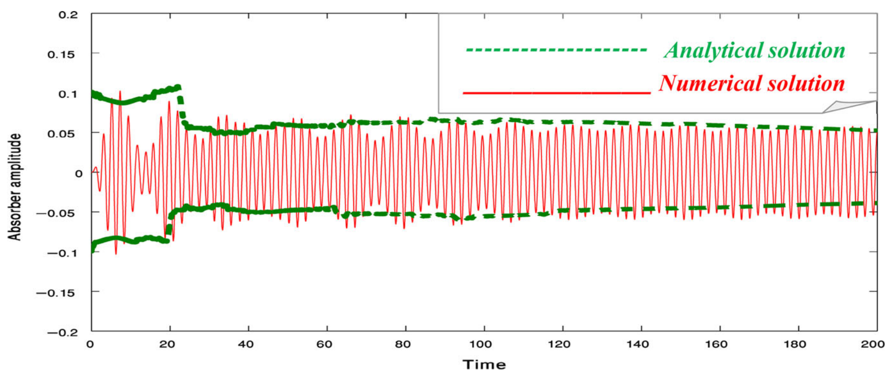

Fig. 4 Comparison between the analytical and numerical solutions of the absorber

4.2 Frequency response curve with the detuning parameter (σ_1)

The effects of different parameters were investigated by solving the frequency response equations (31) and (34) numerically as shown in Figs. 5, 6, 7, 8, 9, 10, 11 and 12. The frequency response curves (FRC) consist of two branches: the solid line represents the stable solution while the dashed line represents the unstable solution, and accordingly their multi-valued solutions. Figure 5 represents the main figure for the selected practical case ($a_1 \neq 0$, $a_2 \neq 0$). The steady-state amplitudes a_1 and a_2 are presented against detuning parameter σ_1. The jump phenomenon is illustrated for example in Fig. 5 (a_1 & σ_1). As σ_1 is reduced from a value corresponding to the point A, the amplitude

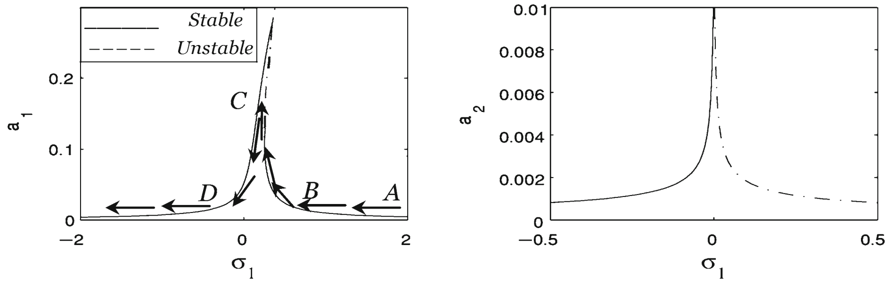

Fig. 5 Effects of the detuning parameter σ_1

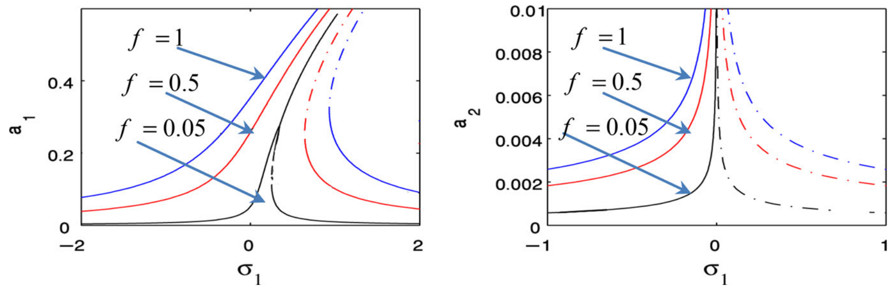

Fig. 6 Effects of increasing the external forcing amplitude f (a_1 and a_2 against σ_1)

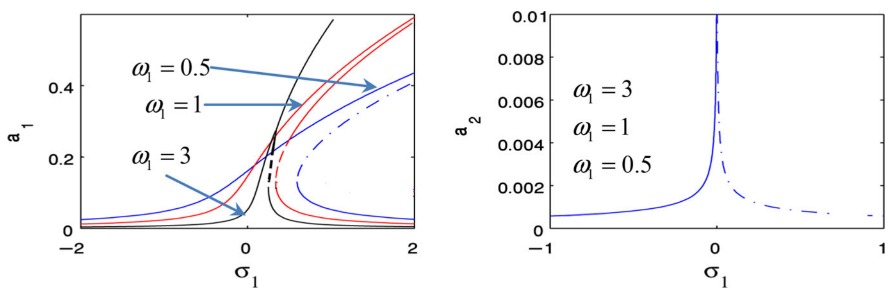

Fig. 7 Effects of the natural frequency ω_1 (a_1 and a_2 against σ_1)

remains to approximately zero until the point B is reached. As σ_1 is decreased further, a jump takes place from the point B to the point C. Then, as σ_1 is decreased further, the amplitude decreases slowly to the point D.

4.2.1 *Effect of the external forcing amplitude f and the natural frequency ω_1*

From Figs. 6 and 7, the steady-state amplitude of the main system (a_1) is monotonic increasing function of the external excitation force amplitude f and the natural frequency ω_1. For decreasing ω_1, the curve that is bent to the right leads to the occurrence of the jump phenomena.

Also, Fig. 6 shows that the steady-state amplitude of the absorber (a_2) is monotonic increasing function of the external excitation force amplitude f but for increasing values of natural frequency ω_1, the steady-state amplitude of the absorber (a_2) is trivial due to the occurrence of saturation phenomena as shown in Fig. 7.

4.2.2 *Effect of the damping coefficients μ and ζ_1*

Figures 8 and 9 illustrated that the steady-state amplitude of the main system (a_1) is decreased when the damping coefficients μ and ζ_1 are increased.

Furthermore, Fig. 8 presented that the steady-state amplitude of the absorber (a_2) is monotonic decreasing function of the damping coefficients μ but monotonic decreasing function for of the damping coefficients ζ_1 as shown in Fig. 9.

4.2.3 *Effect of the linear parameter α*

From Fig. 10, the steady-state amplitude of the main system (a_1) is shifted to right and the steady-state

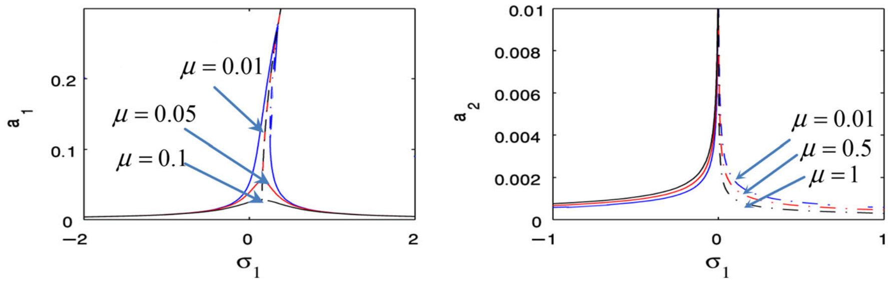

Fig. 8 Effects of increasing the damping coefficient μ (a_1 and a_2 against σ_1)

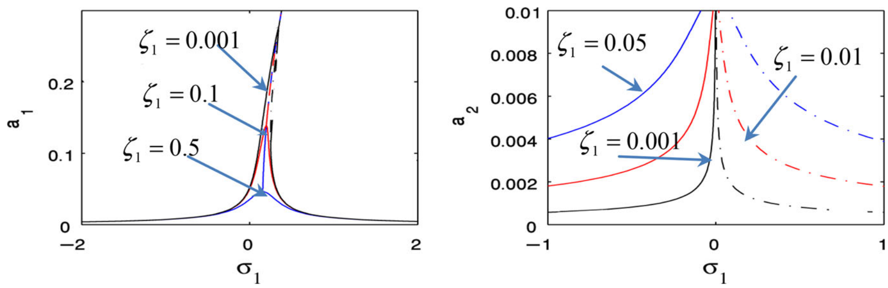

Fig. 9 Effects of increasing the damping coefficient ζ_1 (a_1 and a_2 against σ_1)

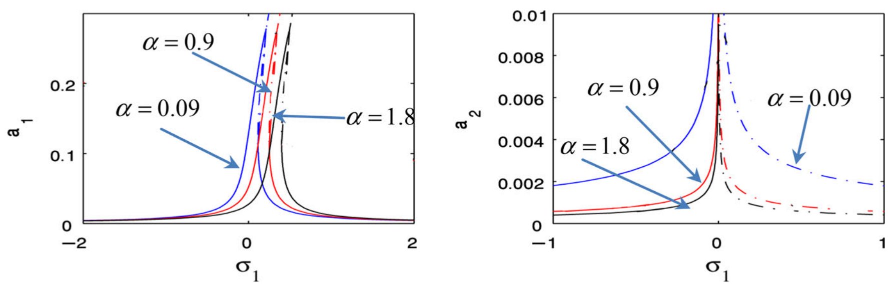

Fig. 10 Effects of the linear parameter α (a_1 and a_2 against σ_1)

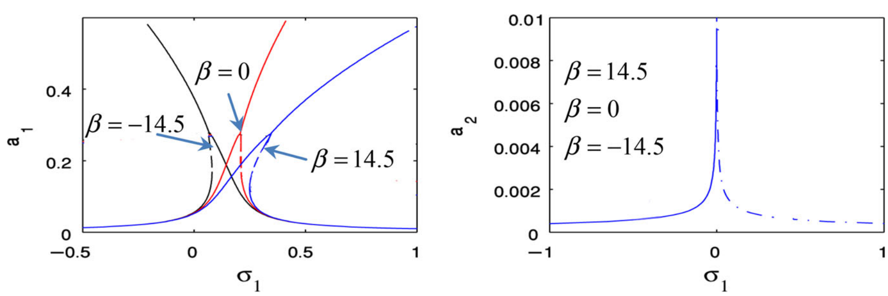

Fig. 11 Effects of the nonlinear parameter β (a_1 and a_2 against σ_1)

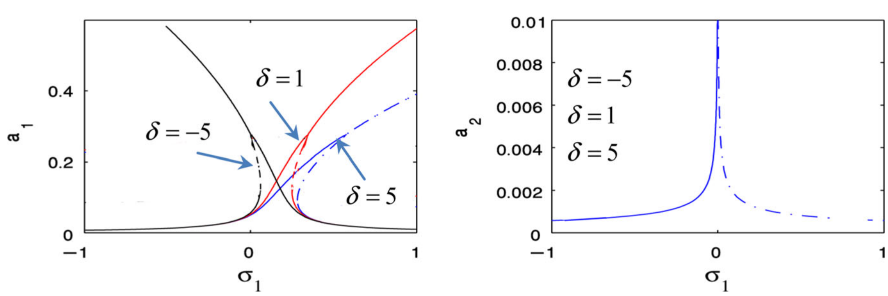

Fig. 12 Effects of the nonlinear parameter δ (a_1 and a_2 against σ_1)

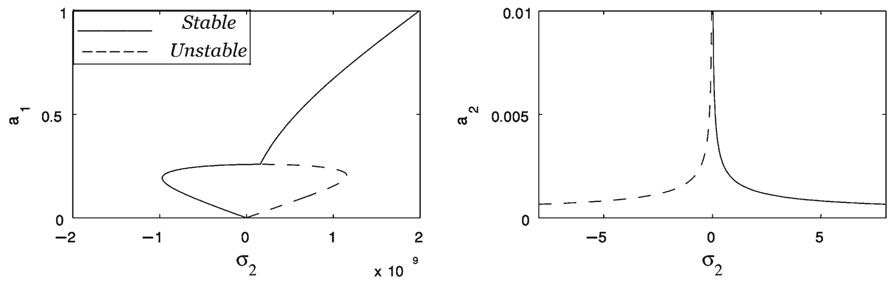

Fig. 13 Effects of the detuning parameter σ_2

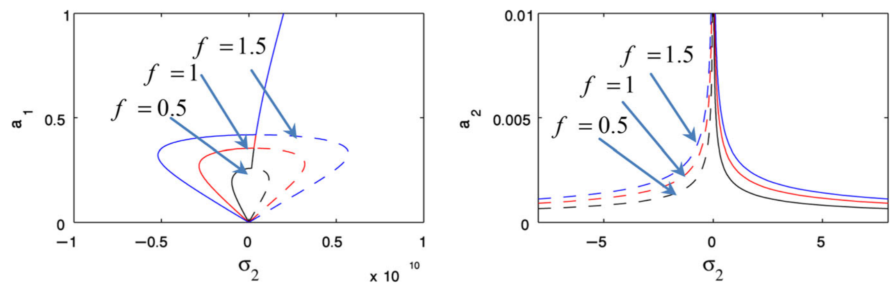

Fig. 14 Effects of increasing the external forcing amplitude f (a_1 and a_2 against σ_2)

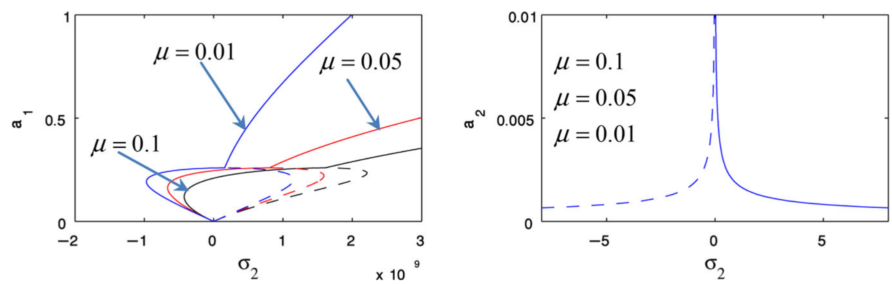

Fig. 15 Effects of increasing the damping coefficient μ (a_1 and a_2 against σ_2)

amplitude of the absorber (a_2) is decreased when the linear parameter α is increased.

4.2.4 *Effect of the nonlinear parameters β and δ*

Figures 11 and 12 show that for increase or decrease of the nonlinear parameters β and δ produce either hard or soft spring, respectively, that leads to occurrence of jump phenomena for the steady-state amplitude of the main system (a_1) but for increasing values of β and δ, the steady-state amplitude of the absorber (a_2) is trivial due to the occurrence of saturation phenomena

4.3 Frequency response curve with the detuning parameter (σ_2)

Figure 13 represents the main figure for the selected practical case ($a_1 \neq 0$, $a_2 \neq 0$). The steady-state amplitudes a_1 and a_2 are presented against detuning parameter σ_2. The steady-state amplitude a_1 has a continuous symmetrical curve at $\sigma_2 = 0$. But the steady-state amplitude a_2 will separate at $\sigma_2 = 0$ and consists of two branches of the curve.

From Figs. 14, 15, 16, 17, 18, 19, and 20, the steady-state amplitude of the main system (a_1) is monotonic increasing function of the external excitation force amplitude f and the damping coefficient ζ_1 with the increasing in the separation distance for nonzero solution at $\sigma_2 = 0$. But the steady-state amplitude of the main system (a_1) is monotonic decreasing function of the damping coefficient μ, natural frequency ω_1, the linear parameter α, and nonlinear parameters β and δ with the decreasing in the separation distance for nonzero solution at $\sigma_2 = 0$.

Moreover, the steady-state amplitude of the absorber (a_2) is increased when the external excitation force amplitude f and the damping coefficient ζ_1 are increased. Also, for increasing values of natural frequency ω_1 and the linear parameter α, the steady-state ampli-

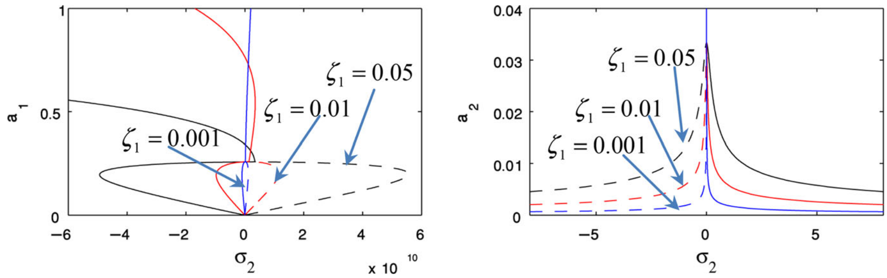

Fig. 16 Effects of increasing the damping coefficient ζ_1 (a_1 and a_2 against σ_2)

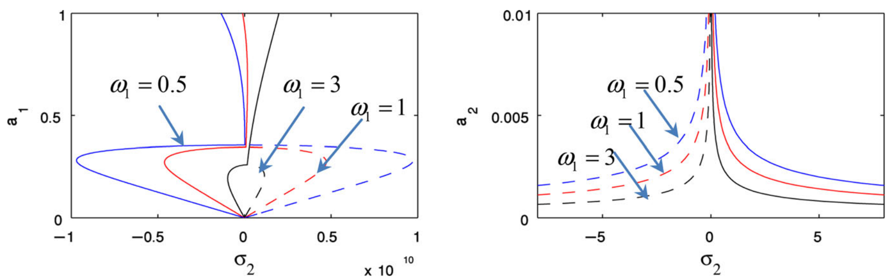

Fig. 17 Effects of the natural frequency ω_1 (a_1 and a_2 against σ_2)

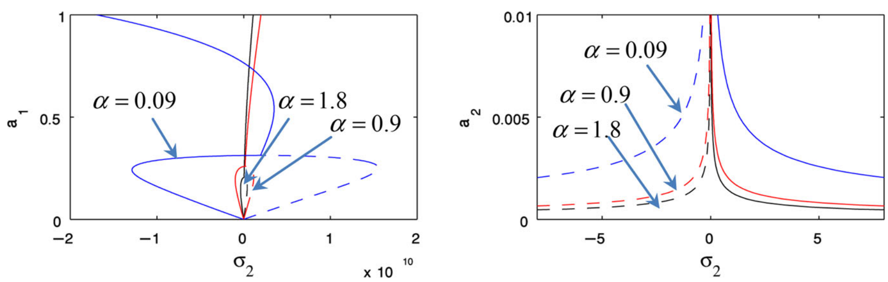

Fig. 18 Effects of the linear parameter α (a_1 and a_2 against σ_2)

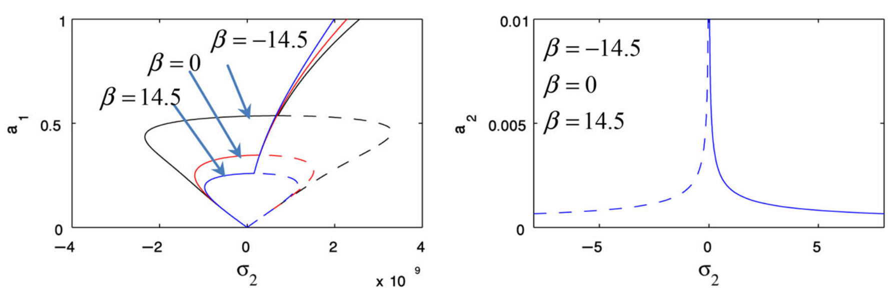

Fig. 19 Effects of the nonlinear parameter β (a_1 and a_2 against σ_2)

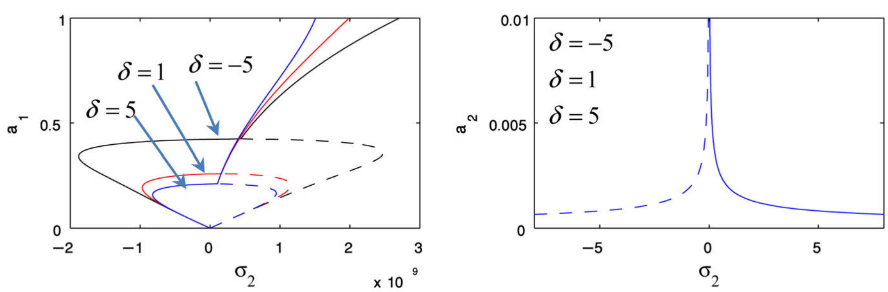

Fig. 20 Effects of the nonlinear parameter δ (a_1 and a_2 against σ_2)

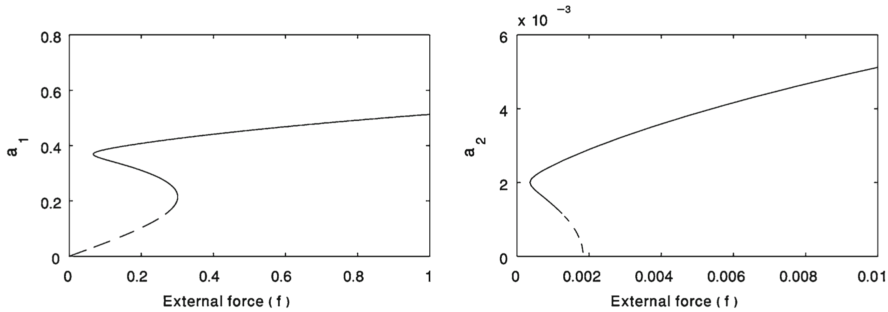

Fig. 21 Force-amplitude response curves (a_1 and a_2 against f)

tude of the absorber (a_2) is decreased. But for increasing values of μ, β, and δ, the steady-state amplitude of the absorber (a_2) is trivial due to the occurrence of saturation phenomena

4.4 Force response curve with the external force (f)

In Fig. 21, the force response curve of both the main system and the absorber is presented at the presence of 1:1 internal resonance. Moreover, from the histories of a_1 and a_2 as the excitation amplitude f slowly is increased from zero and 0.002 respectively.

5 Conclusions

The absorber which is the one of the most common methods of passive vibration control has been presented to reduce the vibration of the nonlinear system. This vibrating system subject to external excitation force has been studied for the primary resonance in the presence of 1:1 internal resonance. The method of multiple scales perturbation is applied to solve the nonlinear differential equations describing the system up to the first-order approximation near primary and internal resonance case of this system. Then, from the analytical analysis, the frequency response equations are obtained and the conditions of stability are considered. The numerical analysis is investigated the performance of the control law and the effectiveness of the absorber. The analyses are revealed that

The primary resonance case ($\Omega \cong \omega_1$) is one of the worst resonance cases for the nonlinear dynamical system and we must avoid it.

The type of the passive control (absorber) is very suitable to suppress the high amplitude vibration of the nonlinear system when the natural frequency of the absorber is properly tuned to the excitation frequency ($\Omega \cong \omega_2$).

The efficiency of the absorber to control the vibrating system (E_a) is about 800. Where the steady-state amplitude of the main system (u) can be reduced to 99.875 % of the main value.

The steady-state amplitude of the main system (u) with the detuning parameter (σ_1) is monotonic increasing functions of f, ω_1, and monotonic decreasing functions of μ, ζ_1.

The steady-state amplitude of the main system (u) with the detuning parameter (σ_2) is monotonic increasing functions of f, ζ_1, and monotonic decreasing functions of μ, ω_1, α, β, δ.

These results indicate the good effectiveness of the suggested absorber compared to the PPF controller in Ref. [2] regarding the amplitude reduction.

5.1 Comparison with published work

Literature review [1] illustrated the numerical and experimental studies of four types active controllers. The results of the dynamical system show that the nonlinear saturation controller (NSC) and the positive position feedback (PPF) are the most effective for assumed conditions of the considered plant. Ref. [2] studied the coupled nonlinear dynamical system with PPF controller subjected to external force at simultaneous primary and internal resonance case. They found that the steady-state amplitude of the system (u) is reduced to about 98.5 % from its value without control. The effectiveness of the PPF controller for this system is about $E_a = 625$ and they found that all predictions from analytical solutions are in good agreement with the numerical simulation.

In this paper, the same nonlinear dynamical system in Refs. [1,2] is investigated with replacing the controller system by a passive controller instated of active controller. With the suggested passive controller, the steady-state amplitude of the main system (u) is reduced to 99.875 % from its value without absorber. This means that the effectiveness of the absorber E_a is about 800 for the main system (u) at the simultaneous primary and internal resonance case. These results indicate the good effectiveness of the suggested absorber in decreasing the vibration of the amplitude for the main system (u) compared to the PPF controller in Ref. [2]

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